Throughput and Delay in Random Wireless Networks: 1-D Mobility is Just As Good As 2-D

James Mammen and Devavrat Shah
Departments of Electrical Engineering and Computer Science
Stanford University, Stanford, CA 94305
{jmammen, devavrat}@stanford.edu

I. Introduction and Model

Grossglauser and Tse [3] introduced a mobile random network model where each node moves independently on a unit disk according to a stationary uniform distribution and showed that throughput of $\Theta(1)$ is achievable. It was shown in [2] that the delay associated with this throughput scales as $\Theta\left(\sqrt{n}/v(n)\right)$ where $v(n)$ is the velocity of the nodes. In [1], a more restricted mobility model was considered where each node moves along a randomly chosen great circle on the unit sphere. It was shown that even with this 1-D restriction on mobility, constant throughput scaling is achievable. In this paper, we study the delay scaling for a mobile network with this 1-D mobility restriction and show, somewhat surprisingly, that the delay scales as $\Theta\left(\sqrt{n}/v(n)\right)$. Thus, this particular mobility restriction *does not* affect the throughput or the delay performance of the network.

Our random network model consists of n nodes on a sphere of unit area. Associated with each node is a great circle chosen independently according to a uniform distribution. Each node moves independently according to a 1-D Brownian motion on its great circle. We denote the velocity of the nodes by $v(n)$ and assume that the velocity scales down with n. Each node picks another node at random as its destination. We assume the *relaxed protocol model* where a transmission from node i to node j is successful if, for any other node k that is transmitting simultaneously, $d(k,j) \geq (1+\Delta)d(i,j)$ for $\Delta > 0$, where $d(i,j)$ is the distance between nodes i and j. Each node can transmit at W bits/second if this constraint is satisfied. Time is slotted for transmission and the duration of the time-slots do not scale with n.

Definition of throughput: A throughput $\lambda > 0$ is said to be feasible/achievable if every node can send at a rate of λ bits/second to its chosen destination. We denote by $T(n)$, the maximum feasible throughput scaling with high probability (*whp*) over the random network configurations due to the random configurations of the great circles and node mobility.

Definition of delay: The delay of a packet in a network is the time it takes the packet to reach the destination after it leaves the source. The average packet delay for a network with n nodes, $D(n)$, is obtained by averaging over all packets, all source-destination pairs, and all random network configurations.

II. Throughput and delay with 1-D mobility

Let $gc(i)$ denote the great circle of node $i \in \{1,\ldots,n\}$. Let z_{ij} denote a point of intersection of $gc(i)$ and $gc(j)$ and let $\mathcal{C}_{ij}$ denote a circle on the sphere centered at z_{ij} with diameter $1/\sqrt{n}$. We say that node i and j are neighbors at time t if they are both in $\mathcal{C}_{ij}$ at time t. It is possible that at some time a node i may not have any neighbors.

Consider the following scheme, which is an adaptation of Scheme 2 in [2]. In this scheme each time-slot is divided into two sub-slots A and B and each node becomes active in a time-slot at random with some constant probability. If an active node has one or more neighbors it chooses any one at random and does the following. In sub-slot A, it transmits a packet intended for its destination to the randomly chosen node which acts as a relay or destination. In sub-slot B, it transmits a packet it is carrying as a relay or source, destined for the randomly chosen neighboring node. If a packet transmission is not successful it is retransmitted at the next opportunity.

Theorem 1. *The above scheme has* $T(n) = \Theta(1)$.

To determine the delay for this scheme, note that only $\Theta(1/n)$ of the packets belonging to any source-destination (S-D) pair reach their destination in a single hop (which happens when S transmits directly to D). Thus, most of the packets reach their destination via a relay node, where the delay has two components: (i) *hop-delay*, which is constant, independent of n, and (ii) *mobile-delay*, which is the time the packet spends at the relay while it is moving. Hence the delay scaling $D(n)$ is determined by the mobile-delay.

For any S-D pair, each of the remaining $n-2$ nodes acts as a relay node. Each node keeps a separate queue for each S-D pair. Thus the mobile-delay is the average delay at such a queue. By symmetry, all such queues at all relay nodes are identical. It turns out that each relay queue is a GI/GI/1-FCFS queue for which the average delay can be bounded using the means and the variances of the inter-arrival time and the service time. These turn out to be of the same order as the mean and variance of the inter-meeting time at the starting position of two nodes performing independent symmetric random walks on a one-dimensional torus of size $\sqrt{n}$. Using this method, we obtain the following result.

Theorem 2. *The above scheme has delay* $D(n) = \Theta\left(\sqrt{n}/v(n)\right)$.

Thus the throughput and delay scaling for the mobile random network with unrestricted mobility and the one with 1-D mobility turn out to be the same. This is a consequence of the fact that although the models seem to be very different, as far as the network properties determining throughput and delay are concerned, they are essentially the same.

References

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