Maximal Matching Scheduling is Good Enough

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Abstract—

In high-speed switches the Input Queued(IQ) architecture is very popular due to its low memory-bandwidth requirement compared to the Output Queued (OQ) switch architecture which is extremely desirable in terms of performance but requires very high memory-bandwidth. In the past decade researchers and industry people have been trying hard to £nd good scheduling algorithm for IQ switches. The two main performance criteria for a scheduling algorithm are: (i) throughput, and (ii) delay. There has been a lot of research done to £nd throughput of scheduling algorithms, but a little has been known about delay performance of algorithms. This paper mainly studies the delay properties of a class of scheduling algorithms known as ***maximal matching*** **algorithms.**

It has been known that ***Maximum weight matching*****(MWM) scheduling algorithm provides the maximum possible throughput, also denoted as 100% throughput [1], [2], [4]. The delay bounds for MWM algorithm, and a suite of approximations of MWM algorithm, are known under Bernoulli i.i.d. traf£c. Unfortunately there are two problems: (i) MWM and its approximations are not implementable, and (ii) delay bounds are very weak compared to the known theoretical lower bounds that can be obtained in terms of performance of an OQ switch.**

On the other end of spectrum lies simple ***maximal matching*** **algorithm like iSLIP [5] which is implemented in commercially available routers. In [4] it was shown that all** ***maximal matching*** **scheduling algorithms are stable at** ***speedup*** **of 2. But nothing is known about their delay performance. In this paper, we obtain bounds on all** ***maximal matching*** **scheduling algorithm running at speedup 2 when traf£c is Bernoulli i.i.d. Interestingly, these bounds match the theoretical lower bound very closely and much better than the bounds on MWM. In particular, we show that any CIOQ switch running at speedup 2 with maximal matching schedule as at most 5 times longer queue-sizes on average compared to the OQ switch under Bernolli i.i.d. traf£c. This suggests that under assumption of traf£c being** ***independent enough*****, no switch can do better than a simple maximal matching algorithm running at speedup 2. This provides the £rst theoretical support to "iSLIP can provide QoS".**

We would like to note that any IQ switch architecture that needs to support OQ switch must have speedup 2 as shown in [9], [10]. The algorithms proposed in [9], [10] are very complex compared to algorithms like iSLIP.

I. Introduction

Over the past decade the IQ switch architecture has become dominant in high speed switching. This is mainly due to the fact that the memory bandwidth of its packet buffers is very low compared to that of an output-queued or a shared-memory architecture.

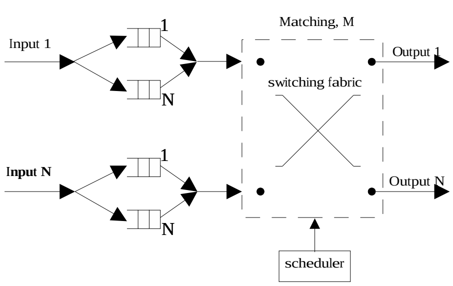

Fig. 1. Logical structure of an input-queued cell switch

Fig. 1 shows the logical structure for an input-queued (IQ) switch. Suppose that time is slotted so that at most one packet can arrive at each input in one time slot. Packets arriving at input i and destined for output j are buffered in a "virtual output queue" (VOQ), denoted here by VOQ_{ij}. The use of virtual output queues avoids performance degradation due to the head-of-line blocking phenomenon [8]. Let the average cell arrival rate at input i for output j be λ_{ij}. The incoming traf£c is called *admissible* if $\sum_{i=1}^{N} \lambda_{ij} < 1$, and $\sum_{j=1}^{N} \lambda_{ij} < 1$. Let the maximum load on any row/column be λ which can be precisely de£ned as

$$\lambda = \max_k \{ \sum_{j=1}^{N} \lambda_{kj}, \sum_{i=1}^{N} \lambda_{ik} \}.$$

For simplicity we will assume that for all i, j, $\lambda_{i.} = \sum_{k=1}^{N} \lambda_{ik} = \lambda$ and $\lambda_{.j} = \sum_{k=1}^{N} \lambda_{kj} = \lambda$, that is, all row and columns are loaded with equal overall load though individual λ_{ij} can be different. When traf£c is Bernoulli i.i.d. a packet arriving at input i is for output j with probability λ_{ij} independent of past and other inputs. For notational simplicity in the rest of the paper for any vector/matrix X we denote $X_{i.} = \sum_k X_{ik}$ and $X_{.j} = \sum_k X_{kj}$.

We assume that packets are switched from inputs to outputs by a crossbar fabric. When switching unicast traf£c [1], this fabric imposes the following constraint: in each time slot, at most one packet may be removed from each input and at most one packet may be transferred to each output. We would like to note that *speedup* of c in an IQ switch means that in one packet time-slot c scheduling cycles can take place. Thus in an IQ switch with speedup 2 by the time one packet can arrive to an input, at most 2 packets can go to output from this input. At speedup higher than 1, this imposes requirement of having queues at output side too. We will call such switches as Combined Input Output Queued (CIOQ) switches.

To perform well, an $N \times N$ input-queued switch requires a good packet scheduling algorithm for determining which inputs to connect with which outputs in each time slot. It is well-known that the crossbar constraint makes the switch scheduling problem a matching problem in an $N \times N$ weighted bipartite graph. The weight of the edge connecting input i to output j is often chosen to be some quantity that indicates the level of congestion; for example, queue-lengths or the ages of packets.

A matching for this bipartite graph is a valid schedule for the switch. Note that a valid matching can be seen as a permutation of the N outputs. In this paper we will use the words *schedule*, *matching* and *permutation* interchangeably. A matching of particular importance for this paper is the Maximum Weight Matching algorithm (MWM). Given a weighted bipartite graph, the

[1] We do not consider multicast traf£c in this paper.

MWM £nds that matching whose weight is the highest. For example, Figure 2 shows a weighted bipartite graph and one valid schedule (or matching). In [3], [7] bounds on average delay for MWM and a suite of approximations of MWM-like scheduling algorithms under Bernoulli i.i.d. traf£c were obtained. Roughly speaking these bounds are $O(N^2)$ with respect to switch size N. As we shall show the average delay/queue-size for an OQ switch under Bernoulli i.i.d. traf£c are $O(N)$. Thus theoretically known bounds on MWM are quite weak compared to known lower bounds. Further, these algorithms are too complicated to implement due to various reasons. In [11], many easy to implement algorithms were proposed. These algorithms have provable 100% throughput and simulations show good delay property but nothing is known about their performance relative to an OQ switch's performance.

In past very interesting research has been done to show the possibility of emulating performance of an OQ switch in a Combined Input Output Queued(CIOQ) switch at a constant speedup [9], [10]. They showed that it is not possible to emulate OQ switches at speedup lower than 2. They proposed algorithms to emulate OQ switch performance at speedup 2 and 4. These algorithms are stable-marriage type algorithms which do packet-by-packet scheduling. This makes it very dif£cult to implement as it requires to process a lot of information at every time slot. Thus though theoretically performance is guaranteed to be as good as an OQ switch, it is not feasible to implement.

The main question remains: is it possible to have simple scheduling algorithm that run at speedup 2 and almost do as good as an OQ switch. We answer this question in af£rmative sense in this paper by showing that maximal matching algorithm has such properties.

A maximal matching scheduling algorithm obtains a *maximal* schedule, that is it has the following property: *no pair of input and output which are not connected under the schedule do not have packets for each other*. The other implication of this algorithm is that, if an input i has packets for output j, then at least one of the input i and output j is connected in any maximal matching. One such maximal matching algorithm is known as iSLIP [5]. A version of iSLIP is known to be implemented in commercially available routers. These maximal matching algorithms do not require any kind of weight information like queue-length, age of cell etc. or any urgency information required in OQ switch emulation. This makes it very simple to implement in IQ switches. Hence it is important to know the theoretical properties of such maximal matching algorithms.

It is known that any maximal matching algorithm with speedup 2– every cell-time switch schedules packets 2 times– provides 100% throughput under any admissible traf£c [4], [12]. But nothing is known about the average queue size or delays.

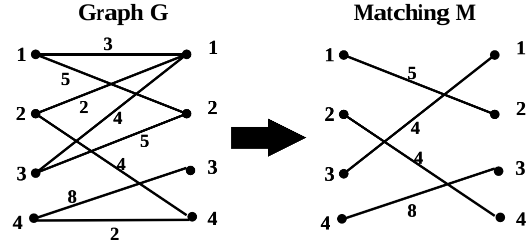

Fig. 2. Example of weighted bipartite graph and its maximum weight matching.

In this paper, we study the average queue size under maximal matching algorithm at speedup 2 under any admissible Bernoulli i.i.d. traf£c. We obtain upper bound on the average queue size. The proof technique used here involves two main tools: (i) a Lyapunov function based argument (similar to [7]), and (ii) a novel combinatorial argument.

Using simple (and novel as far as our knowledge) argument that an OQ switch is the best any switch, and in particular a CIOQ switch at speedup 2, can do with outgoing line rate being 1, we obtain lower bound on the average delay.

Interestingly, the upper bound and lower bounds obtained are of the same order(see section III-II for details). This suggests that the average queue size of a CIOQ switch operating under maximal matching at speed up 2 is very similar to that of the queue sizes in an OQ switch.

The following is a strong implication: a CIOQ switch running at speedup 2 with iSLIP as scheduling algorithm performs as good as an OQ switch in terms of (i) throughput, and (ii) average delay.

The rest of the paper is organized as follows: In section II and section III we obtain lower bound and upper bound respectively for the average queue size of a CIOQ switch with any maximal matching scheduling algorithm at speedup 2 with admissible Bernoulli i.i.d. arrival traf£c. In section IV we present the conclusion.

II. Average Queue size: Lower bound

To obtain lower bound on the net queue size, we turn our attention to the Output Queued(OQ) switches. In OQ switches, there is no unnecessary queuing due to switch scheduling. The queues build up in OQ switch due to the arrival traf£c. Since in all switch architecture, IQ, CIOQ or OQ, this queuing has to occur as in all switches output line runs at speed 1, the average queue size on in OQ switch is the lower bound on all switching architecture.

Before proceeding further consider the following notations: Let $Y_i(t)$ be queue size at output i at time t. Let $Y(t) = \sum_i Y_i(t)$ be the total queue size of the OQ switch at time t. We claim the following theorem for the average queue size in OQ switch:

Theorem 1. *Under Bernoulli i.i.d. traf£c with arrival-rate matrix Λ with $\lambda < 1$ as the average load on each input and output node. Then,*

$$E[Y(t)] \quad = \quad \frac{N\lambda}{2(1-\lambda)} \tag{1}$$

Proof. In OQ switch consider one particular output, say 1. Everytime, it receives packets from possibly N inputs. At each input the packets arrive for output 1 according to N independent Bernoulli i.i.d. processes with rates $\lambda_{11}, \ldots, \lambda_{N1}$. The output 1 serves out 1 packet at a time. Thus the output 1 is a deterministic server of rate 1. For large N, we can approximate this queue at output 1 by an $M/D/1$ queue with arrival process being Poisson of rate $\lambda = \sum_i \lambda_{i1}$ (use of Binomial-Poisson approximation). By Pollaczek-Khinchine formula [6], we obtain that, the average queue size for such an $M/D/1$ queue is

$$\begin{aligned} E[Y_1(t)] &= \frac{\lambda ES^2}{2(1-\lambda ES)} \\ &= \frac{\lambda}{2(1-\lambda)} \end{aligned} \quad (2)$$

where, $ES^2 = ES = 1$ as the deterministic service of rate 1. Since all outputs are symmetric, we obtain the similar is true to $E[Y_i(t)]$ for all i. Thus, we obtain,

$$\begin{aligned} E[Y(t)] &= E[\sum_{i=1}^{N} Y_i(t)] \\ &= NE[Y_1(t)] \\ &= \frac{N\lambda}{2(1-\lambda)} \end{aligned} \quad (3)$$

This proves our claimed Theorem 1. □

III. AVERAGE QUEUE SIZE: UPPER BOUND

In this section we obtain an upper bound on the average queue size for CIOQ switch running with any maximal matching running at speedup 2. We need to bound queue sizes at inputs as well as outputs. First we obtain bounds on the queue sizes at input side. We will use this bound to obtain the bounds on queue sizes at output. This will give us bound on the net queue-size of such a CIOQ switch.

Let $Q(t) = [Q_{ij}(t)]$ be the $N \times N$ matrix of N^2 VOQs in the switch at input side. Let the net queue size at input side be denoted as $Z(t) = \sum_{i,j} Q_{ij}(t)$. We state the following theorem:

Theorem 2. *Let a CIOQ switch running at speedup* 2 *and uses any maximal matching algorithm for scheduling. Let the traf£c be Bernoulli i.i.d. with rate matrix* Λ *with row/column load of* $\lambda < 1$. *In equilibrium, the average of* $Z(t)$ *can be bounded as*

$$E[Z(t)] \leq \frac{\lambda N}{(1-\lambda)} \quad (4)$$

Proof. To prove this theorem we use Lyapunov function technique used in [3], [7]. Consider the following Lyapunov function

$$f(t) = \langle Q(t), C(t) \rangle \triangleq \sum_{i,j} Q_{ij}(t) C_{ij}(t)$$

where,

$$C(t) = MQ(t) + Q(t)M,$$

where, $M = [m_{ij}]$ with $m_{ij} = 1$ for all i, j. In other words, $C(t) = [C_{ij}(t)]$ then $C_{ij}(t) = \sum_k Q_{ik}(t) + \sum_l Q_{lj}(t)$.

We would like to £nd the $f(t+1) - f(t)$ given the $Q(t)$. This in turn will imply the desired bound. Consider the following:

$$f(t+1) - f(t) = \sum_{i,j} Q_{ij}(t+1) C_{ij}(t+1) - Q_{ij}(t) C_{ij}(t)$$

By queuing equation, $Q_{ij}(t+1) = [Q_{ij}(t) + A_{ij}(t) - D_{ij}(t)]^+$. Let us neglect the non-negativity: neglecting it means at most $Q_{ij}(t+1)$ can be 1 due to an arrival after departure. Since at most N arrivals per time slot are possible this requires us to add N to the bound on $Z(t)$ to overcome the error introduced by neglecting it. As we will see, the £nal bound on $Z(t)$ is higher and hence we can neglect this. Thus, effectively we obtain,

$$Q_{ij}(t+1) = Q_{ij}(t) + A_{ij}(t+1) - D_{ij}(t) \quad (5)$$

Similarly, for $C_{ij}(t)$ we obtain,

$$\begin{aligned} C_{ij}(t+1) &= C_{ij}(t) + A_{i.}(t+1) + \\ &\quad A_{.j}(t+1) - D_{i.}(t) - D_{.j}(t) \end{aligned} \quad (6)$$

Using (5)-(6), we obtain,

$$\begin{aligned} f(t+1) - f(t) = \sum_{i,j} \{ &[Q_{ij}(t) + A_{ij}(t+1) - D_{ij}(t)] \times \\ &[C_{ij}(t) + A_{i.}(t+1) + A_{.j}(t+1) - D_{i.}(t) - D_{.j}(t)]\} \\ &\quad - Q_{ij}(t)C_{ij}(t) \\ = \sum_{i,j} \{ &Q_{ij}(t)[A_{i.}(t+1) + A_{.j}(t+1) - D_{i.}(t) - D_{.j}(t)] \\ + \ &C_{ij}(t)(A_{ij}(t+1) - D_{ij}(t+1)) + [A_{ij}(t+1) \\ - \ &D_{ij}(t+1)] \times [A_{i.}(t+1) + A_{.j}(t+1) \\ &\quad - D_{i.}(t) - D_{.j}(t)]\} \end{aligned} \quad (7)$$

It will be clear later that our main goal is to bound $E[f(t+1) - f(t)|Q(t)]$. Hence in (7) we bound the expected change for each term. First we bound the last term in (7) on the right hand side as follows:

$$(A_{ij}(t+1) - D_{ij}(t+1)) \leq A_{ij}(t), \quad \text{and}$$

$$(A_{i.}(t+1) + A_{.j}(t+1) - D_{i.}(t) - D_{.j}(t)) \leq 2$$

Hence,

$$\begin{aligned} \sum_{i,j} &[A_{ij}(t+1) - D_{ij}(t+1)][A_{i.}(t+1) + A_{.j}(t+1) \\ &\quad - D_{i.}(t) - D_{.j}(t)] \leq 2\lambda N \end{aligned} \quad (8)$$

For the other two terms, note that by simple algebraic manipulation

$$\begin{aligned} &\sum_{i,j} Q_{ij}(t)[A_{i.}(t+1) + A_{.j}(t+1) - D_{i.}(t) - D_{.j}(t)] \\ = &\sum_{i,j} C_{ij}(t) \times (A_{ij}(t+1) - D_{ij}(t+1)) \end{aligned} \quad (9)$$

By property of maximal matching, if $Q_{ij}(t) > 0$ then either input i is matched to some output or output j is matched to some input. Hence at every schedule, $D_{i.}(t) + D_{.j}(t)$ is at least 1 given $Q_{ij}(t) > 0$. At speedup 2 scheduling happens twice. Hence every time $D_{i.}(t) + D_{.j}(t) \geq 2$. Before proceeding further, we would like to note that, if $Q_{ij}(t) = 1$ and it is served in the £rst phase of the scheduling, then it may become 0, and hence have to neglect its effect as $Q_{ij}(t) > 0$ in the above analysis. But note that, only N of them can have this effect and such N add only N to the net queue size in the effect and hence by adding this additional N to the upper bound, we can neglect this effect (there are other ways to do this, but it being non-important point we do not discuss here in detail). From above discussion, we obtain that if $Q_{ij}(t) > 0$ then $D_{i.}(t) + D_{.j}(t) \geq 2$. Along with equations (8)-(9) and this observation and noticing that $E[Q_{ij}(t)(A_{i.}(t+1) + A_{.j}(t+1))|Q(t)] = Q_{ij}(t)(2\lambda)$ due to the traf£c being independent in time, we obtain,

$$\begin{aligned} E[f(t+1) - f(t)|Q(t)] &\leq 2\sum_{i,j} Q_{ij}(t)[2\lambda - 2] + 2\lambda N \\ &= 4(\lambda - 1)Z(t) + 2\lambda N \qquad (10) \end{aligned}$$

Consider the following (using (10)):

$$\begin{aligned} E[f(t+1)] &= E[E[f(t+1) - f(t) + f(t)|Q(t)]] \\ &= E[E[f(t+1) - f(t)|Q(t)]] + E[f(t)] \\ &\leq E[4(\lambda - 1)Z(t) + 2\lambda N] + E[f(t)] \quad (11) \end{aligned}$$

Summing the equation (11) for $t = 0, \ldots, T$, we obtain that,

$$\begin{aligned} E[f(T+1)] &\leq 4(\lambda - 1)\sum_{t=0}^{T} E[Z(t)] + 2\lambda N(T+1) \\ &\quad + E[f(0)] \qquad (12) \end{aligned}$$

Since the system is stable $\lim_{T\to\infty} \frac{E[f(T+1)]}{T+1} = 0$ and assuming system starts with £nite queue size, we have $\lim_{T\to\infty} \frac{E[f(0)]}{T+1} = 0$ Further, since our system $Q(t)$ is a discrete time Markov process and it being aperiodic and irreducible, it is ergodic. For Bernoulli i.i.d. traf£c by result of stability [12] we know that $Q(t)$ has bounded average (also other bounded moments). By ergodic theorem $\lim_{T\to\infty} \frac{\sum_{t=0}^{T} Z(t)}{T+1} = E[Z(t)]$ in equilibrium. Hence the bounded moment imply the convergence in L^1 also. Hence we £nally obtain using (12) that in equilibrium,

$$\begin{aligned} E[Z(t)] &= \lim_{T\to\infty} \frac{\sum_{t=0}^{T} Z(t)}{T+1} \\ &\leq \frac{\lambda N^2}{2(1-\lambda)} \qquad (13) \end{aligned}$$

If we consider the neglected N term, we obtain bound as,

$$E[Z(t)] \leq \frac{\lambda N}{(1-\lambda)} \qquad (14)$$

Thus we have proved the desired results as claimed in the statement of Theorem 2. □

Note: we note that the above bound is a bit weaker and can be tightened. Though qualitatively the result does not change.

The above theorem bounds the queue sizes formed at the input side in the CIOQ switch. In such an CIOQ switch the bound on queue size at the input side is important as the packets transferred to output side will get transferred without problem. But we need to prove concretely that this is the case. We use a novel technique based on the idea of "leaky bucket constrainted" traf£c to prove this.

We £rst consider the following scenario: packets arrive at a queue with a deterministic server running at rate 1 according to arrival process $A(t)$. The number of packets arrived during any time period $[s, t]$, denote as $A(s, t)$, obey the leaky bucket constraint:

$$A(s,t) \leq \sigma(t - s) + \rho, \quad \forall 0 \leq s < t \qquad (15)$$

where, $\sigma < 1$ and ρ any positive £xed integer. The following is a well-known result:

Lemma 1. *For such a leaky-bucket constrainted queue, the maximum queue-size is bounded above by ρ.*

Proof. Since $\sigma < 1$ the queue is stable. By Lindley's equation, the queue size at time t is,

$$Q(t) = \max_{0\leq s\leq t} \{A(s,t) - (t - s)\}.$$

By replacing $A(s,t) \leq \sigma(t-s)+\rho$, we obtain that the maximum is ρ for $t = s$. This proves the lemma. □

To obtain bound on the queue-size on the output side, we consider the following "virtual" OQ switch. This OQ switch has two sources of packet arrivals: (i) packets arriving according to the original Bernoulli i.i.d. arrival process, and (ii) the packets queued up on the input-side of the original CIOQ switch. The way to view the (i) as the "σ" part of the arrival process as in (15) and (ii) as the "ρ" part of the arrival process. In Lemma 1 the queue-sizes are formed due to "ρ" part. In this case, part (i) also forms some queue-size, which from Theorem 1 are on average:

$$E[Q^1] = \frac{N\lambda}{2(1-\lambda)} \qquad (16)$$

The additional queue-size formed due to (ii) are exactly same as the queue-size on input side, which on average is bounded above by Theorem 2, as

$$E[Q^2] = \frac{N\lambda}{(1-\lambda)} \qquad (17)$$

Thus, by argument similar to Lemma 1, we obtain that the net queue size for such a virtual OQ is bounded above by:

$$\begin{aligned} E[Q^v_{OQ}] &\leq E[Q^1] + E[Q^2] \\ &= \frac{N\lambda}{2(1-\lambda)} + \frac{N\lambda}{(1-\lambda)} \\ &= \frac{3N\lambda}{2(1-\lambda)} \qquad (18) \end{aligned}$$

Next notice the following two facts: (i) by construction of the arrival process, the virtual OQ switch has more packets coming to output side compared to the original CIOQ switch, and (ii) the output side of CIOQ switch and this virtual OQ switch are identical.

This leads us to the following theorem:

Theorem 3. *Let the net queue size at the input and output of an CIOQ switch be $R(t)$ at time t. Let the arrival traf£c be Bernoulli i.i.d. with traf£c rate matrix Λ with row and column loaded with load λ as described before. Then under any* *maximal matching with speedup* 2 *the average of* $R(t)$ *can be bounded above as*

$$E[R(t)] \leq \frac{5N\lambda}{2(1-\lambda)}.$$

Proof. From above discussion, Theorem 2 and Theorem 1the statement of the theorem follows immediately. □

A. Discussion

B. CIOQ v/s OQ

From Theorem 3 and Theorem 1 the comparison suggests:

$$E[Y(t)] \quad \leq \quad E[R(t)] \quad \leq \quad 5E[Y(t)] \qquad (19)$$

Thus, the CIOQ switch with maximal matching algorithm (for example, iSLIP) has at most 5 times larger queues compared to an OQ switch on average under Bernoulli i.i.d. traf£c.

B.1 CIOQ, MWM and OQ

As proved in [3], [7], the bound on average queue-size under MWM are of order $O\left(\frac{\lambda N^2}{(1-\lambda)}\right)$. As noted above the bound for maximal-speedup 2-CIOQ or OQ of order $O\left(\frac{\lambda N}{(1-\lambda)}\right)$. Thus bound on MWM is $O(N)$ times weaker than that for CIOQ or OQ. This suggests the weakness of bounds on MWM. Certainly this does not suggest that actually MWM is too poor, but shows the weakness of currently known results.

B.2 Arrival distribution

To prove above results we assumed the Bernoulli i.i.d. assumption. We can extend this to any re-generative/markovian traf£c and can obtain similar qualitative results. That is, the average queue-sizes for a CIOQ switch running at speedup 2 with maximal matching is of the same order as that of for an OQ switch.

IV. Conclusions

In this paper we have analyzed the average queue size for a CIOQ switch running at speedup 2 under any maximal matching under Bernoulli i.i.d. arrival traf£c. We obtained upper bound and lower bound for the average queue sizes. These bounds differ from each other by a constant factor (independent of switch size or other parameters). This suggests that CIOQ switch with any maximal matching at speedup 2 performs as good as any switch architecture (including OQ architecture) for any Bernoulli i.i.d. arrival traf£c. Thus simple algorithms like iSLIP on a CIOQ switch running at speedup 2 performs as good as an OQ switch in terms of throughput and delay.

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